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# FUZZY SOFT GRADATION OF OPENNESS

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## ABSTRACT
In this paper the concept of fuzzy soft gradation of openness is introduced and using this concept a new definition of fuzzy soft topology is obtained and some of its properties are studied.

**KEYWORDS**: Fuzzy topology, Gradation of openness, Fuzzy soft topology, Fuzzy soft gradation of openness.

## INTRODUCTION
There are many mathematical tools for dealing with uncertainties. The most appropriate theories for dealing with uncertainties are fuzzy set theory introduced by Zadeh,L.A.[13] and soft set theory introduced by Molodstov[7]. In 1968, Chang,C.L.[1] defined fuzzy topology and later in 1976, Lowen,R.[4] defined fuzzy topology in a different way. In 1992, Hazra,R.N., Samantha,S.K., and Chattopadhyay,K.C.[2] introduced the concept of gradation of openness and gave a new definition of fuzzy topology. In 2001, Maji et al.[5] initiated the concept of fuzzy soft set which is a combination of fuzzy set and soft set. Tanay and Kandemir [12] introduced topological structure of fuzzy soft sets and gave a introductory base for further study on this concept following the way of Chang and PazarVarol and Aygun [8] defined fuzzy soft topology in Lowen's sense. In section II of this paper, preliminary definitions regarding fuzzy set, soft set, fuzzy soft set are given. In section III of this paper , the definition of fuzzy topology through the concept of gradation of openness introduced by Hazra,R.N., Samantha,S.K., and Chattopadhyay,K.C.[2] is extended to fuzzy soft topology by introducing the concept of fuzzy soft gradation of openness.

## PRELIMINARY DEFINITIONS
Throughout this paper, X denotes initial universe and E denotes the set of parameters for the universe X.

**Definition 2.1[13]**
A fuzzy set in X is a map $f : X \to [0, 1] = I$. The family of fuzzy sets in X is denoted by $I^X$. Following are some basic operations on fuzzy sets. For the fuzzy sets f and g in X,

(1) $f = g \Leftrightarrow f(x) = g(x)$ for all $x \in X$.
(2) $f \leq g \Leftrightarrow f(x) \leq g(x)$ for all $x \in X$.
(3) $(f \vee g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.
(4) $(f \wedge g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$.
(5) $f^C(x) = 1 - f(x)$ for all $x \in X$. Here $f^C$ denotes the complement of f.
(6) For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X.
  (i) $(\vee_{\lambda\in\Lambda} f_\lambda)(x) = \vee_{\lambda\in\Lambda} f_\lambda(x)$
  (ii) $(\wedge_{\lambda\in\Lambda} f_\lambda)(x) = \wedge_{\lambda\in\Lambda} f_\lambda(x)$
(7) For any $\alpha \in I$, the constant fuzzy set $\boldsymbol{\alpha}$ in X is a fuzzy set in X defined by $\boldsymbol{\alpha}(x) = \alpha$ for all $x \in X$ .
  **0** denotes null fuzzy set in X and **1** denotes universal fuzzy set in X.

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**Definition 2.2[1]**
A fuzzy topological space is a pair (X, $\tau$) where X is a non empty set and $\tau$ is a family of fuzzy sets on X satisfying the following properties :

(1) the constant functions $\mathbf{0}$ and $\mathbf{1}$ belong to $\tau$.
(2) f, g $\in\tau$ implies $f \wedge g \in\tau$
(3) $f_\lambda \in \tau$ for each $\lambda\in\Lambda$ implies $\vee_{\lambda\in\Lambda} f_\lambda \in\tau$.

Then $\tau$ is called a fuzzy topology on X. Every member of $\tau$ is called fuzzy open. g is called fuzzy closed in (X, $\tau$) if $g^C \in \tau$.

**Definition 2.3[2]**
Let X be a nonempty set. A mapping $G : I^X \to I$ is said to be a gradation of openness on X iff the following conditions are satisfied :

($G_1$) $\mathcal{G}(\mathbf{0}) = \mathcal{G}(\mathbf{1}) = 1$

($G_2$) $\mathcal{G}(f_i) > 0$ for $i = 1, 2, \ldots, m \Rightarrow \mathcal{G}\left(\bigwedge_{i=1}^{m} f_i\right) > 0$

($G_3$) $\mathcal{G}(f_\lambda) > 0$ for $\lambda \in \Lambda \Rightarrow \mathcal{G}(\vee_{\lambda\in\Lambda} f_\lambda) > 0$

The pair (X, $\mathcal{G}$) is called a gradation space.

**Definition 2.4[2]**
Let (X, $\mathcal{G}$) be a gradation space. Then the fuzzy topology on X induced by $\mathcal{G}$ is given by $\tau(\mathcal{G}) = \{ f\in I^X / \mathcal{G}(f) > 0\}$.

**Definition 2.5[2]**
Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two gradations of openness on X. Then $\mathcal{G}_1 \geq \mathcal{G}_2$ if $\mathcal{G}_1(f) \geq \mathcal{G}_2(f)$ for all $f \in I^X$.

**Definition 2.6[2]**
Let (X, $\mathcal{G}_1$) and (Y, $\mathcal{G}_2$) be two gradation spaces. Then a map $\theta : X \to Y$ is called

(1) a gradation preserving map, if $\mathcal{G}_2(f) \leq \mathcal{G}_1(\theta^{-1}(f))$ for each $f \in I^Y$.
(2) a strongly gradation preserving map, if $\mathcal{G}_2(f) = \mathcal{G}_1(\theta^{-1}(f))$ for each $f \in I^Y$.
(3) a weakly gradation preserving map, if $\mathcal{G}_2(f) > 0 \Longrightarrow \mathcal{G}_1(\theta^{-1}(f)) > 0$ for each $f \in I^Y$.

**Definition 2.7[7]**
Let $A \subseteq E$. A soft set $f_A$ over X is a mapping from E to P(X) i.e., $f_A : E \to P(X)$ where P(X) is the power set of X.

**Definition 2.8[9]**
Let $A \subseteq E$. A fuzzy soft set $\tilde{f}_A$ over X is a mapping from E to $I^X$ i.e., $\tilde{f}_A : E \to I^X$ where $\tilde{f}_A(e) \neq \mathbf{0}$ if $e \in A \subseteq E$ and $\tilde{f}_A(e) = \mathbf{0}$ if $e \notin A$. The family of fuzzy soft sets over X is denoted by FS(X, E).

**Definition 2.9[9]**
The fuzzy soft set $\tilde{f}_\phi \in FS(X, E)$ is called null fuzzy soft set denoted by $\tilde{\mathbf{0}}$ if for all $e \in E$, $\tilde{f}_\phi(e) = \mathbf{0}$.

**Definition 2.10[9]**
The fuzzy soft set $\tilde{f}_E \in FS(X, E)$ is called universal fuzzy soft set denoted by $\tilde{\mathbf{1}}$ if for all $e \in E$, $\tilde{f}_E(e) = \mathbf{1}$.

**Definition 2.11[9]**
Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. $\tilde{f}_A$ is called a fuzzy soft subset of $\tilde{g}_B$ if $A \subseteq B$ and $\tilde{f}_A(e) \leq \tilde{g}_B(e)$ for every $e \in E$ and we write $\tilde{f}_A \,\tilde{\subseteq}\, \tilde{g}_B$.

**Definition 2.12[9]**
Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. $\tilde{f}_A$ and $\tilde{g}_B$ are said to be equal denoted by $\tilde{f}_A = \tilde{g}_B$ if $\tilde{f}_A \,\tilde{\subseteq}\, \tilde{g}_B$ and $\tilde{g}_B \,\tilde{\subseteq}\, \tilde{f}_A$.

**Definition 2.13[9]**

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Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. The union of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \vee \tilde{g}_B(e)$ for all $e \in E$ where $C = A \cup B$. Here we write $\tilde{h}_C = \tilde{f}_A \,\tilde{\cup}\, \tilde{g}_B$.

**Definition 2.14[9]**

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. The intersection of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \wedge \tilde{g}_B(e)$ for all $e \in E$ where $C = A \cap B$. Here we write $\tilde{h}_C = \tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B$.

**Definition 2.15[9]**

Let $\tilde{f}_A \in FS(X, E)$. The complement of $\tilde{f}_A$ denoted by $\tilde{f}_A^C$ is a fuzzy soft set defined by $\tilde{f}_A^C(e) = 1_X - \tilde{f}_A(e)$ for every $e \in E$. Clearly $(\tilde{f}_A^C)^C = \tilde{f}_A$, $\tilde{\mathbf{1}}^C = \tilde{\mathbf{0}}$ and $\tilde{\mathbf{0}}^C = \tilde{\mathbf{1}}$

**Definition 2.16[12]**

A fuzzy soft topological space is a triple $(X, E, \tilde{\tau})$ where X is a nonempty set, E is a parameter set and $\tilde{\tau}$ is a family of fuzzy soft sets over X satisfying the following properties :

(1) $\tilde{\mathbf{0}}, \tilde{\mathbf{1}} \in \tilde{\tau}$

(2) $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ then $\tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B \in \tilde{\tau}$

(3) If $\tilde{f}_{A_i} \in \tilde{\tau}\ \forall i \in \Lambda$ then $\tilde{\cup}_{\lambda \in \Lambda} \tilde{f}_{A_i} \in \tilde{\tau}$

Then $\tilde{\tau}$ is called a fuzzy soft topology over X. Every member of $\tilde{\tau}$ is called fuzzy soft open. $\tilde{g}_B$ is called fuzzy soft closed in $(X, E, \tilde{\tau})$ if $\tilde{g}_B^C \in \tilde{\tau}$.

**Definition 2.17[10]**

Let FS(X, E) and FS(Y, K) be the families of all fuzzy soft sets over (X, E) and (Y, K) respectively. Let $u : X \to Y$ and $p : E \to K$ be two functions. Then $\tilde{f}_{up}$ is called a fuzzy soft mapping from X to Y and denoted by $\tilde{f}_{up} : FS(X, E) \to FS(Y, K)$.

(1) Let $\tilde{f}_A \in FS(X, E)$, then the image of $\tilde{f}_A$ under the fuzzy soft mapping $\tilde{f}_{up}$ is the fuzzy soft set over Y defined by $\tilde{f}_{up}(\tilde{f}_A)$, where

$$\tilde{f}_{up}(\tilde{f}_A)(k)(y) = \vee_{x \in u^{-1}(y)} \left( \vee_{e \in p^{-1}(k) \cap A} \tilde{f}_A(e) \right)(x) \quad \text{if } u^{-1}(y) \neq \phi,\ p^{-1}(k) \cap A \neq \phi$$
$$= 0 \text{ in Y, otherwise.}$$

(2) Let $\tilde{g}_B \in FS(Y, K)$, then the preimage of $\tilde{g}_B$ under the fuzzy soft mapping $\tilde{f}_{up}$ is the fuzzy soft set over X defined by $(\tilde{f}_{up})^{-1}(\tilde{g}_B)$ where

$$(\tilde{f}_{up})^{-1}(\tilde{g}_B)(e)(x) = \begin{cases} \tilde{g}_B(p(e))(u(x)), & \text{for } p(e) \in B \\ 0 \text{ in X}, & \text{otherwise} \end{cases}$$

**Theorem 2.18[10]**

Let $\tilde{f}_A \in FS(X, E)$, $(\tilde{f}_{A_i})_{i \in \wedge} \in FS(X, E)$ and $\tilde{g}_B \in FS(Y, K)$, $(\tilde{g}_{B_i})_{i \in \wedge} \in FS(Y, K)$, where $\Lambda$ is an index set.

(1) If $\tilde{f}_{A_1} \,\tilde{\subseteq}\, \tilde{f}_{A_2}$ then $\tilde{f}_{up}(\tilde{f}_{A_1}) \,\tilde{\subseteq}\, \tilde{f}_{up}(\tilde{f}_{A_2})$

(2) If $\tilde{g}_{B_1} \,\tilde{\subseteq}\, \tilde{g}_{B_2}$ then $\tilde{f}_{up}^{-1}(\tilde{g}_{B_1}) \,\tilde{\subseteq}\, \tilde{f}_{up}^{-1}(\tilde{g}_{B_2})$

(3) $\tilde{f}_{up}(\tilde{\cup}_{i \in \wedge} \tilde{f}_{A_i}) = \tilde{\cup}_{i \in \wedge} \tilde{f}_{up}(\tilde{f}_{A_i})$

(4) $\tilde{f}_{up}(\tilde{\bigcap}_{i\in\Lambda}\tilde{f}_{A_i}) = \tilde{\bigcap}_{i\in\Lambda}\tilde{f}_{up}(\tilde{f}_{A_i})$

(5) $\tilde{f}_{up}^{-1}(\tilde{\bigcup}_{i\in\Lambda}\tilde{g}_{B_i}) = \tilde{\bigcup}_{i\in\Lambda}\tilde{f}_{up}^{-1}(\tilde{g}_{B_i})$

(6) $\tilde{f}_{up}^{-1}(\tilde{\bigcap}_{i\in\Lambda}\tilde{g}_{B_i}) = \tilde{\bigcap}_{i\in\Lambda}\tilde{f}_{up}^{-1}(\tilde{g}_{B_i})$

(7) $\tilde{f}_{up}^{-1}(\tilde{\mathbf{1}}$ over Y$) = \tilde{\mathbf{1}}$ over X, $\tilde{f}_{up}^{-1}(\tilde{\mathbf{0}}$ over Y$) = \tilde{\mathbf{0}}$ over X

(8) $\tilde{f}_{up}(\tilde{\mathbf{0}}$ over X$) = \tilde{\mathbf{0}}$ over Y, $\tilde{f}_{up}(\tilde{\mathbf{1}}$ over X$) = \tilde{\mathbf{1}}$ over Y

**Definition 2.19[10]**

Let (X, E, $\tilde{\tau}_1$) and (Y,K, $\tilde{\tau}_2$) be two fuzzy soft topological spaces. A fuzzy soft mapping $\tilde{f}_{up}$ : (X,E, $\tilde{\tau}_1$) $\to$ (Y,K, $\tilde{\tau}_2$) is called fuzzy soft continuous if $(\tilde{f}_{up})^{-1}(\tilde{g}_B) \in \tilde{\tau}_1$ for all $\tilde{g}_B \in \tilde{\tau}_2$.

## FUZZY SOFT GRADATION OF OPENNESS

**Definition 3.1**

Let X be an initial universe and E be a set of parameters. A mapping $\tilde{\mathcal{G}}$ : FS(X, E) $\to$ I is said to be a fuzzy soft gradation of openness over (X, E) iff the following conditions are satisfied.

(FSG 1) $\tilde{\mathcal{G}}(\tilde{\mathbf{0}}) = (\tilde{\mathbf{1}}) = 1$

(FSG 2) $\tilde{\mathcal{G}}(\tilde{f}_{A_i}) > 0$ for i = 1, 2, ... , m $\Rightarrow \tilde{\mathcal{G}}(\tilde{\bigcap}_{i=1\text{ to }m}\tilde{f}_{A_i}) > 0$

(FSG 3) $\tilde{\mathcal{G}}(\tilde{f}_{A_\lambda}) > 0$ for $\lambda\in\Lambda \Rightarrow \tilde{\mathcal{G}}(\tilde{\bigcup}_{\lambda\in\Lambda}\tilde{f}_{A_\lambda}) > 0$

The triple (X, E, $\tilde{\mathcal{G}}$) is called a fuzzy soft gradation space.

**Definition 3.2**

Let (X, E, $\tilde{\mathcal{G}}$) be a fuzzy soft gradation space. The fuzzy soft topology over (X, E) induced by $\tilde{\mathcal{G}}$ is given by $\tilde{\tau}(\tilde{\mathcal{G}}) = \{\tilde{f}_A \in$ FS(X, E) / $\tilde{\mathcal{G}}(\tilde{f}_A) > 0\}$.

**Definition 3.3**

Let $\tilde{\mathcal{G}}_1$ and $\tilde{\mathcal{G}}_2$ be two fuzzy soft gradations of openness over (X, E). Then $\tilde{\mathcal{G}}_1 \geq \tilde{\mathcal{G}}_2$ if $\tilde{\mathcal{G}}_1(\tilde{f}_A) \geq \tilde{\mathcal{G}}_2(\tilde{f}_A)$ for all $\tilde{f}_A \in$ FS(X, E).

**Definition 3.4**

Let (X, E, $\tilde{\mathcal{G}}_1$) and (Y, K, $\tilde{\mathcal{G}}_2$) be two fuzzy soft gradation spaces. Let u : X $\to$ Y and p : E $\to$ K be two functions. Then a map $\tilde{f}_{up}$ : FS(X, E) $\to$ FS(Y, K) is called

(1) a fuzzy soft gradation preserving map if $\tilde{\mathcal{G}}_2(\tilde{f}_A) \leq \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right)$ for each $\tilde{f}_A \in$ FS(Y, K).

(2) a fuzzy soft strongly gradation preserving map if $\tilde{\mathcal{G}}_2(\tilde{f}_A) = \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right)$ for each $\tilde{f}_A \in$ FS (Y, K).

(3) a fuzzy set weakly gradation preserving map if $\tilde{\mathcal{G}}_2(\tilde{f}_A) > 0 \Rightarrow \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right) > 0$ for each $\tilde{f}_A \in$ FS (Y, K).

**Theorem 3.5**

Let $\tilde{f}_{up}$ : FS(X, E) → FS(Y, K) be a map where u : X → Y and p : E → K be two functions. Let $\tilde{\mathcal{G}}_1$ be a fuzzy soft gradation of openness over (X, E). Then the largest fuzzy soft gradation of openness $\tilde{\mathcal{G}}_2$ over (Y, K) which makes $\tilde{f}_{up}$ : (X, E, $\tilde{\mathcal{G}}_1$) → (Y, K, $\tilde{\mathcal{G}}_2$) a fuzzy soft gradation preserving map is given by

$\tilde{\mathcal{G}}_2(\tilde{f}_A) = \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right)$ for each $\tilde{f}_A \in$ FS (Y, K).

**Proof :**

For $\tilde{f}_A \in$ FS (Y, K), define $\tilde{\mathcal{G}}_2(\tilde{f}_A) = \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right)$.

**Claim 1 :**

(FSG 1) $\tilde{\mathcal{G}}_2\,(\tilde{\mathbf{0}} \text{ over } Y) = \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{\mathbf{0}} \text{ over } Y)\right) = \tilde{\mathcal{G}}_1(\tilde{\mathbf{0}} \text{ over } X) = 1$

$\tilde{\mathcal{G}}_2\,(\tilde{\mathbf{1}} \text{ over } Y) = \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{\mathbf{1}} \text{ over } Y)\right) = \tilde{\mathcal{G}}_1(\tilde{\mathbf{1}} \text{ over } X) = 1$

(FSG 2) $\tilde{\mathcal{G}}_2(\tilde{f}_{A_i}) > 0$ for i = 1, 2, ..., m

$\Rightarrow \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_{A_i})\right) > 0$ for i = 1, 2, ..., m

$\Rightarrow \tilde{\mathcal{G}}_1\left(\tilde{\bigcap}_{i=1 \text{ to } m}\,(\tilde{f}_{up})^{-1}(\tilde{f}_{A_i})\right) > 0$

$\Rightarrow \tilde{\mathcal{G}}_1\,(\tilde{f}_{up})^{-1}\left(\tilde{\bigcap}_{i=1 \text{ to } m}\,(\tilde{f}_{A_i})\right) > 0$

$\Rightarrow \tilde{\mathcal{G}}_2\left(\tilde{\bigcap}_{i=1 \text{ to } m}\,\tilde{f}_{A_i}\right) > 0$

(FSG 3) $\tilde{\mathcal{G}}_2(\tilde{f}_{A_\lambda}) > 0$ for $\lambda \in \Lambda$

$\Rightarrow \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_{A_\lambda})\right) > 0$ for $\lambda \in \Lambda$

$\Rightarrow \tilde{\mathcal{G}}_1\left(\tilde{\bigcup}(\tilde{f}_{up})^{-1}(\tilde{f}_{A_\lambda})\right) > 0$

$\Rightarrow \tilde{\mathcal{G}}_1\,(\tilde{f}_{up})^{-1}\left(\tilde{\bigcup}_{\lambda\in\Lambda}\,(\tilde{f}_{A_\lambda})\right) > 0$

$\Rightarrow \tilde{\mathcal{G}}_2\left(\tilde{\bigcup}_{\lambda\in\Lambda}\,\tilde{f}_{A_\lambda}\right) > 0$

**Claim 2 :**

$\tilde{\mathcal{G}}_2$ is the largest fuzzy soft gradation of openness over (Y, K) such that $\tilde{f}_{up}$ : (X, E, $\tilde{\mathcal{G}}_1$) and (Y, K, $\tilde{\mathcal{G}}_2$) is a fuzzy soft gradation preserving map. Suppose $(\tilde{\mathcal{G}}_2)'$ is a fuzzy soft gradation of openness over (Y, K) such that $\tilde{f}_{up}$ : (X, E, $\tilde{\mathcal{G}}_1$) → (Y, K, $(\tilde{\mathcal{G}}_2)'$) is a fuzzy soft gradation preserving map, then $(\tilde{\mathcal{G}}_2)'(\tilde{f}_A) \le \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right)$ for each $\tilde{f}_A \in$ FS(Y, K). Therefore $(\tilde{\mathcal{G}}_2)'(\tilde{f}_A) \le (\tilde{\mathcal{G}}_2)(\tilde{f}_A)$ by the definition of $\tilde{\mathcal{G}}_2$. Hence $\tilde{\mathcal{G}}_2$ is the largest fuzzy soft gradation of openness over (Y, K) such that $\tilde{f}_{up}$ : (X, E, $\tilde{\mathcal{G}}_1$) → (Y, K, $\tilde{\mathcal{G}}_2$) is a fuzzy soft gradation preserving map.

**Theorem 3.6**

$\tilde{f}_{up}$ : (X, E, $\tilde{\mathcal{G}}_1$) → (Y, K, $\tilde{\mathcal{G}}_2$) is a fuzzy soft weakly gradation preserving map iff $\tilde{f}_{up}$ : (X, E, $\tilde{\tau}(\tilde{\mathcal{G}}_1)$) → (Y, K, $\tilde{\tau}(\tilde{\mathcal{G}}_2)$) is fuzzy soft continuous.

**Proof :**

Assume $\tilde{f}_{up}$ is a weakly fuzzy soft gradation preserving map.

Let $\tilde{f}_A \in \tilde{\tau}(\tilde{g}_2)$.

$\tilde{\mathcal{G}}_2\,(\tilde{f}_A) > 0 \Rightarrow \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right) > 0$

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$\Rightarrow (\tilde{f}_{up})^{-1}\tilde{f}_A \in \tilde{\tau}(\tilde{\mathcal{G}}_1)$

Therefore $\tilde{f}_{up}$ is fuzzy soft continuous.

Conversely, assume $\tilde{f}_{up}$ is fuzzy soft continuous.

Let $\tilde{\mathcal{G}}_2(\tilde{f}_A) > 0 \Rightarrow \tilde{f}_A \in \tilde{\tau}(\tilde{\mathcal{G}}_2)$

$\Rightarrow (\tilde{f}_{up})^{-1}(\tilde{f}_A) \in \tilde{\tau}(\tilde{\mathcal{G}}_1)$

$\Rightarrow \tilde{\mathcal{G}}_1\left((\tilde{f}_{up})^{-1}(\tilde{f}_A)\right) > 0$

Therefore $\tilde{f}_{up}$ is a weakly fuzzy soft gradation preserving map.

## CONCLUSION

In this paper the concepts of fuzzy soft gradation of openness and fuzzy soft topology induced by fuzzy soft gradation of openness are introduced and some basic properties regarding these concepts are proved.

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